Q1

Both upper and lower bounds of m are needed so we will first need both the upper and lower bounds of s and t - with lots of numbers around, keep organised by using a table.
Use the general rule "half down" for lower bounds and "half up" for upper bounds.

Value	Lower bound	Upper bound
s	**3.465** (3.47 - 0.005)	**3.475** (3.47 + 0.005)
t	**8.1315** (8.132 - 0.0005)	**8.1325** (8.132 + 0.0005)

Either bound for s [1]
Either bound for t [1]

As the calculation is division, for the ***lower*** bound of m we need s (and so $\sqrt{s}$) to be as **small** as possible - i.e. its ***lower*** bound and t to be as **large** as possible - i.e. its ***upper*** bound of t.

$$\text{Lower bound of } m = \frac{\sqrt{3.465}}{8.1325} = 0.228\,890\,383\,...$$

For the **upper** bound of m we'll need the opposite - the **upper** bound of s and the **lower** bound of t.

$$\text{Upper bound of } m = \frac{\sqrt{3.475}}{8.1315} = 0.229\,248\,624\,...$$

Either lower or upper bound for m [1]

So we have ...

$$0.228\,890\,... < m < 0.229\,248\,...$$

Both correct values [1]

Look for the highest degree of accuracy that both the upper and lower bounds can be rounded to such that they are equal.

0.228 890 ... = 0.229 to three significant figures
0.229 248 ... = 0.229 to three significant figures

$\therefore m = 0.229$ to three significant figures - this is the highest degree of accuracy that both the lower and upper bounds for m agree [1]

Q2

2

The question suggests an **upper** bound - we need to find the largest possible average speed for Kirsty and compare this to 80 km/h.
The formula connecting average speed, distance and time is

$$(\text{Average})\ \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

This is a **division** calculation so we will need **distance** to be as **large** as possible - its **upper** bound, and **time** to be as **small** as possible - its **lower** bound.

Upper bound for distance is 4535 m (4530 + 5)
Lower bound for time is 202.5 seconds (205 - 2.5)

For 4535 or 202.5 [1]

Be careful - the distance is given in metres and the time given is in seconds, the value we are comparing to is in kilometres per hour.
Convert distance to kilometres by dividing by 1000 and convert seconds to hours by dividing by 60 twice.
Then apply the formula for average speed.

$$\frac{4535 \div 1000}{202.5 \div 60 \div 60} = 80.6\dot{2}$$

Time and distance conversion [1]
Correct calculation for bound [1]

Compare this to 80 km/h.

Kirsty's average speed could be greater than 80 km/h since the upper bound for her average speed is 80.6 km/h (3 s.f.) [1]

You could answer this question using metres per second and converting the 80 km/h to m/s.

Q3

Both upper and lower bounds (of density) are required.
The formulae for density and volume are

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume (of a cuboid)} = \text{length} \times \text{width} \times \text{height}$$

So we need the upper and lower bounds of the length, width, height and mass of the cuboid.
With lots of values around, keep them organised using a table

Value	Lower bound	Upper bound
length	**13.15** (13.2 - 0.05)	**13.25** (13.2 + 0.05)
width	**15.95** (16.0 - 0.05)	**16.05** (16.0 + 0.05)
height	**21.65** (21.7 - 0.05)	**21.75** (21.7 + 0.05)
mass	**1967.5** (1970 - 2.5)	**1972.5** (1970 + 2.5)

One correct bound [1]

Density is a division calculation so for the **lower** bound of we need the **numerator** to be as **small** as possible - i.e. the **lower** bound of **mass,** and the **denominator** to be as **large** as possible - i.e. the **upper** bound of **volume** (**upper bounds of length**, **width** and **height**).

$$\text{Lower bound of density} = \frac{1967.5}{13.25 \times 16.05 \times 21.75} = 0.425\ 367\ 754\ \ldots$$

Bound for volume [1]
Bound for density [1]

For the **upper** bound of density we need the **numerator** to be as **large** as possible - i.e. the **upper** bound of **mass,** and the **denominator** to be as **small** as possible - i.e. the **lower** bound of **volume** (**lower bounds of length**, **width** and **height**).

$$\text{Upper bound of density} = \frac{1972.5}{13.15 \times 15.95 \times 21.65} = 0.434\ 382\ 850\ \ldots$$

For the **upper** bound of density we need the **numerator** to be as **large** as possible - i.e. the **upper** bound of **mass,** and the **denominator** to be as **small** as possible - i.e. the **lower** bound of **volume** (**lower bounds of length**, **width** and **height**).

$$\text{Upper bound of density} = \frac{1972.5}{13.15 \times 15.95 \times 21.65} = 0.434\ 382\ 850\ \ldots$$

So we have ...

$$0.425\ 367\ \ldots < m < 0.434\ 382\ \ldots$$

Both correct [1]

To answer the question we need to look for the highest degree of accuracy that both the upper and lower bounds can be rounded to such that they are equal.

0.425 367 ... = 0.43 to two significant figures
0.434 382 ... = 0.43 to two significant figures

To two significant figures the density of the wood is 0.43 g/mm^3 - this is the highest degree of accuracy to which both the lower and upper bounds agree [1]

Q4

4

In general, to estimate, round values to one significant figure.

Radius of cylinder = 31 cm ≈ 30 cm
Height of cylinder = 97.5 cm ≈ 100 cm

The formula for the volume of a cylinder is $V = \pi r^2 h$.
π also needs rounding to one significant figure.

$$V = 3 \times (30)^2 \times 100 = 270\ 000\ \text{cm}^3$$

The tank is full so this is also the volume of water in the tank.
The answer is required in litres but we are given that 1000 cm^3 = 1 litre

$$270\ 000\ \text{cm}^3 = \frac{270\ 000}{1\ 000} = 270\ \text{litres}$$

There is approximately 270 litres of water in the tank [1]

Q5

5a

In this question it is only appropriate to round one of the values to one significant figure.
8 is a single digit number and is already 'nice' to work with.

$$3069.25 \text{ miles} \approx 3000 \text{ miles}$$

15.12 is roughly half way between 10 and 20 but rounding to either of those would lead to a broad estimate.
But to two significant figures it is 15 and this is still quick work with mentally.

$$15.12 \text{ miles per hour} \approx 15 \text{ miles per hour}$$

Next work out an estimate for the number of miles Juan completes in a day using $\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$.

$$\therefore \text{Distance (per day)} = \text{Average speed per hour} \times \text{Hours per day}$$
$$\text{Distance (per day)} = 15 \times 8 = 120$$

Using a correctly rounded value [1]

Now estimate the number of days by dividing the total number of miles by the number of miles Juan cycles in a day.

$$\frac{3000}{120} = 25$$

[1]

(You can simplify to make the division quicker: $\frac{3000}{120} = \frac{300}{12} = \frac{50}{2} = 25$).

Juan will take approximately 25 days to complete the race [1]

5b

Juan's average speed has increased (both from the orginal 15.12 mph and our rounded 15 mph) so he will complete the race more quickly – i.e. in less days
Therefore the answer to part (a) will decrease [1]

Q6

6a

Start by rounding the mass of one uranium atom to one significant figure.

$$3.95 \times 10^{-22} \approx 4 \times 10^{-22}$$

[1]

Note we have a mixture of grams and kilograms, so convert to grams.

$$1 \text{ kg} = 1000 \text{ g}$$

The calculation for the number of uranium atoms in 1 kg of uranium will be

$$\text{Number of atoms} = \frac{\text{Mass of uranium}}{\text{Mass of one atom}}$$

Substitute our rounded values in to the formula.

$$\text{Number of atoms} = \frac{1000}{4 \times 10^{-22}}$$

$$\frac{1000}{4 \times 10^{-22}} = 0.25 \times 10^{25}$$

[1]

This value is not in standard form ($A \times 10^n$ *where* $1 \le A < 10$).

$$0.25 \times 10^{25} = 2.5 \times 10^{-1} \times 10^{25} = 2.5 \times 10^{24}$$

There are approximately 2.5×10^{24} atoms in 1 kg of uranium [1]

6b

In the calculation $\frac{1000}{4 \times 10^{25}}$, the numerator was left unchanged but the denominator was increased from its actual value
Therefore the result will be smaller than its real value (dividing by a larger number) so our estimate is an underestimate [1]

Q7

To get the upper bound for pressure you need to divide the upper bound for the force by the lower bound for the area.

The force is rounded to the nearest 5 N so the upper bound is halfway between 345 and 350.

Upper bound for force = 345 + 2.5 = 347.5 N

[1]

To find the lower bound for the area you need to multiply the lower bounds of the lengths.
The lengths are rounded to the nearest 0.1 m.
The lower bound for 2.6 m is halfway between 2.5 and 2.6. The lower bound for 6.4 m is halfway between 6.3 and 6.4.

Lower bound for 2.6 m = 2.6 - 0.05 = 2.55 m
Lower bound for 6.4 m = 6.4 - 0.05 = 6.35 m

Any correct bound [1]

Multiply these together.

Lower bound for area = 2.55 × 6.35 = 16.1925 m^2

[1]

Substitute the upper bound for the force and the lower bound for the area into the formula to find the upper bound for the pressure.

$$\text{Upper bound for pressure} = \frac{347.5}{16.1925} = 21.460552\ldots \text{ N/m}^2$$

[1]

Round to 4 significant figures.

Upper bound for pressure = 21.46 N/m^2 (4sf) [1]

Q8

8

Method 1

Mitul has divided the lower bound of the total mass by the upper bound of the mass of one coin. Therefore he has worked out the lower bound for the number of coins.

Check the lower bound for the total mass of 301 coins by multiplying the lower bound of the mass of one coin by 301.

301 × 8.745 g = 2632.245 g

Multiplying a possible mass by 301 [1]
Answer between 2625 and 2635 [1]

Round this to the nearest 10 g and then divide by 1000 to convert to kilograms.

2632.245 g ≈ 2630 g
2630 ÷ 1000 = 2.63 kg

Mitul may be **incorrect** as **£301** could weigh **2.63 kg** to the nearest 10 g [1]

Method 2

Mitul has divided the lower bound of the total mass by the upper bound of the mass of one coin. Therefore he has worked out the lower bound for the number of coins.

Work out the upper bound for the number of coins by dividing the upper bound for the total mass by the lower bound for the mass of a coin.

2635 g ÷ 8.745 g = 301.3150...

Dividing a possible total mass by a possible mass of a coin [1]
Answer over 301 [1]

The number of coins is in the range 299.8 to 301.3 (1dp).

Mitul may be **incorrect** as there could be **£301** [1]